By the end of grade six, students have mastered the four arithmetic operations with whole numbers, positive fractions, positive decimals, and positive and negative integers; they accurately compute and solve problems. They apply their knowledge to statistics and probability. Students understand the concepts of mean, median, and mode of data sets and how to calculate the range. They analyze data and sampling processes for possible bias and misleading conclusions; they use addition and multiplication of fractions routinely to calculate the probabilities for compound events. Students conceptually understand and work with ratios and proportions; they compute percentages (e.g., tax, tips, interest). Students know about π and the formulas for the circumference and area of a circle. They use letters for numbers in formulas involving geometric shapes and in ratios to represent an unknown part of an expression. They solve one-step linear equations.

Number Sense

1.0 Students compare and order positive and negative fractions, decimals, and mixed numbers. Students solve problems involving fractions, ratios, proportions, and percentages:

1.1 Compare and order positive and negative fractions, decimals, and mixed numbers and place them on a number line.

1.2 Interpret and use ratios in different contexts (e.g., batting averages, miles per hour) to show the relative sizes of two quantities, using appropriate notations (a/b, a to b, $a{:}b$).

1.3 Use proportions to solve problems (e.g., determine the value of N if $4/7 = N/21$, find the length of a side of a polygon similar to a known polygon). Use cross-multiplication as a method for solving such problems, understanding it as the multiplication of both sides of an equation by a multiplicative inverse.

1.4 Calculate given percentages of quantities and solve problems involving discounts at sales, interest earned, and tips.

2.0   Students calculate and solve problems involving addition, subtraction, multiplication, and division:

2.1   Solve problems involving addition, subtraction, multiplication, and division of positive fractions and explain why a particular operation was used for a given situation.

2.2   Explain the meaning of multiplication and division of positive fractions and perform the calculations (e.g., $\frac{5}{8} \div \frac{15}{16} = \frac{5}{8} \times \frac{16}{15} = \frac{2}{3}$).

2.3   Solve addition, subtraction, multiplication, and division problems, including those arising in concrete situations, that use positive and negative integers and combinations of these operations.

2.4   Determine the least common multiple and the greatest common divisor of whole numbers; use them to solve problems with fractions (e.g., to find a common denominator to add two fractions or to find the reduced form for a fraction).

Algebra and Functions

1.0   Students write verbal expressions and sentences as algebraic expressions and equations; they evaluate algebraic expressions, solve simple linear equations, and graph and interpret their results:

1.1   Write and solve one-step linear equations in one variable.

1.2   Write and evaluate an algebraic expression for a given situation, using up to three variables.

1.3   Apply algebraic order of operations and the commutative, associative, and distributive properties to evaluate expressions; and justify each step in the process.

1.4   Solve problems manually by using the correct order of operations or by using a scientific calculator.

2.0   Students analyze and use tables, graphs, and rules to solve problems involving rates and proportions:

2.1   Convert one unit of measurement to another (e.g., from feet to miles, from centimeters to inches).

2.2   Demonstrate an understanding that *rate* is a measure of one quantity per unit value of another quantity.

2.3   Solve problems involving rates, average speed, distance, and time.

3.0 Students investigate geometric patterns and describe them algebraically:

3.1 Use variables in expressions describing geometric quantities (e.g., $P = 2w + 2l$, $A = \frac{1}{2} bh$, $C = \pi d$—the formulas for the perimeter of a rectangle, the area of a triangle, and the circumference of a circle, respectively).

3.2 Express in symbolic form simple relationships arising from geometry.

Measurement and Geometry

1.0 Students deepen their understanding of the measurement of plane and solid shapes and use this understanding to solve problems:

1.1 Understand the concept of a constant such as π; know the formulas for the circumference and area of a circle.

1.2 Know common estimates of π (3.14; $\frac{22}{7}$) and use these values to estimate and calculate the circumference and the area of circles; compare with actual measurements.

1.3 Know and use the formulas for the volume of triangular prisms and cylinders (area of base × height); compare these formulas and explain the similarity between them and the formula for the volume of a rectangular solid.

2.0 Students identify and describe the properties of two-dimensional figures:

2.1 Identify angles as vertical, adjacent, complementary, or supplementary and provide descriptions of these terms.

2.2 Use the properties of complementary and supplementary angles and the sum of the angles of a triangle to solve problems involving an unknown angle.

2.3 Draw quadrilaterals and triangles from given information about them (e.g., a quadrilateral having equal sides but no right angles, a right isosceles triangle).

Statistics, Data Analysis, and Probability

1.0 Students compute and analyze statistical measurements for data sets:

1.1 Compute the range, mean, median, and mode of data sets.

1.2 Understand how additional data added to data sets may affect these computations of measures of central tendency.

1.3 Understand how the inclusion or exclusion of outliers affects measures of central tendency.

1.4 Know why a specific measure of central tendency (mean, median, mode) provides the most useful information in a given context.

2.0   Students use data samples of a population and describe the characteristics and limitations of the samples:

2.1   Compare different samples of a population with the data from the entire population and identify a situation in which it makes sense to use a sample.

2.2   Identify different ways of selecting a sample (e.g., convenience sampling, responses to a survey, random sampling) and which method makes a sample more representative for a population.

2.3   Analyze data displays and explain why the way in which the question was asked might have influenced the results obtained and why the way in which the results were displayed might have influenced the conclusions reached.

2.4   Identify data that represent sampling errors and explain why the sample (and the display) might be biased.

2.5   Identify claims based on statistical data and, in simple cases, evaluate the validity of the claims.

3.0   Students determine theoretical and experimental probabilities and use these to make predictions about events:

3.1   Represent all possible outcomes for compound events in an organized way (e.g., tables, grids, tree diagrams) and express the theoretical probability of each outcome.

3.2   Use data to estimate the probability of future events (e.g., batting averages or number of accidents per mile driven).

3.3   Represent probabilities as ratios, proportions, decimals between 0 and 1, and percentages between 0 and 100 and verify that the probabilities computed are reasonable; know that if P is the probability of an event, 1-P is the probability of an event not occurring.

3.4   Understand that the probability of either of two disjoint events occurring is the sum of the two individual probabilities and that the probability of one event following another, in independent trials, is the product of the two probabilities.

3.5   Understand the difference between independent and dependent events.

Mathematical Reasoning

1.0   Students make decisions about how to approach problems:

1.1   Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.

1.2   Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.

1.3   Determine when and how to break a problem into simpler parts.

2.0   Students use strategies, skills, and concepts in finding solutions:

2.1   Use estimation to verify the reasonableness of calculated results.

2.2   Apply strategies and results from simpler problems to more complex problems.

2.3   Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.

2.4   Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.

2.5   Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.

2.6   Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.

2.7   Make precise calculations and check the validity of the results from the context of the problem.

3.0   Students move beyond a particular problem by generalizing to other situations:

3.1   Evaluate the reasonableness of the solution in the context of the original situation.

3.2   Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.

3.3   Develop generalizations of the results obtained and the strategies used and apply them in new problem situations.